Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name: Mathematical Methods – II

Subject Code:5SC04MBE1 **Branch: M.Sc.(Mathematics)**

Semester: 4 **Date: 18/05/2016** **Time: 2:30 To 5:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Find the extremals of the functional $\int_{x_0}^{x_1}(1+x^2y')y'\,dx$. **(02)**

b. Consider $y:[0,1]\to\mathbb{R}$ to be function $y(x)=x$, find the length of the arc y between the points 0 and 1. **(02)**

c. Find the extremals of the functional $\int_1^2 \frac{x^3}{y'^2}\,dx$. **(02)**

d. Define: Geodesic. **(01)**

Q-2 **Attempt all questions** **(14)**

a. Prove that if the functional $I[y(x)]=\int_{x_1}^{x_2} f(x,y,y')\,dx$ has the extremum value, then the integrand f satisfies the Euler's equation $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$. **(06)**

b. Find the extremal of the functional $I=\int_0^\pi [y'^2-y^2]\,dx$ under the conditions $y(0)=0, y(\pi)=1$ and subject to the constraint $\int_0^\pi y\,dx=1$. **(05)**

c. Find the extremals of the functional $\int_{x_0}^{x_1}\frac{1+y^2}{y'^2}\,dx$. **(03)**

OR

Q-2 **Attempt all questions** **(14)**

a. Prove that **(06)**

$$\int_a^x\int_a^{x_n}\dots\int_a^{x_2} f(x_1)dx_1dx_2\dots dx_n=\frac{1}{(n-1)!}\int_a^x (x-t)^{n-1}f(t)\,dt$$

b. Find a curve passing through the points (x_1,y_1) and (x_2,y_2) which when rotated about the x-axis gives a minimum surface area. **(05)**

c. Find the extremals of the functional $\int_0^{\frac{\pi}{2}} (y^2 + y'^2 - 2xy)\ dx, y(0) = 0, y\left(\frac{\pi}{2}\right) = 0$. **(03)**

Q-3 **Attempt all questions** **(14)**

a. Show that the functional $\int_0^{\pi/2} \left\{2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\} dt$ such that $x(0) = 0$, $x\left(\frac{\pi}{2}\right) = -1, y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$ is stationary for $x = -\sin t\,,\ y = \sin t$. **(05)**

b. Find the integral equation corresponding to the boundary value problem $y''(x) - 5\,y'(x) + 6\,y(x) = 0,\quad y(0) = -5, y'(0) = -19.$ **(05)**

c. Show that $y(x) = 2 - x$ is a solution of the integral equation $\int_0^x e^{x-t}\ y(t)\ dt = e^x + x - 1.$ **(04)**

OR

Q-3 **Attempt all questions**

a. Solve the functional $\int_0^1 (y'' - 2xy)\ dx,\ y(0) = y'(0) = 0$ and $y(1) = y'(1) = 1.$ **(05)**

b. Show that the integral equation $y(x) = \int_0^x (x + t)y(t)\ dt + 1$is equivalent to the differential equation$y''(x) - 2xy'(x) - 3y(x) = 0, y(0) = 1, y'(0) = 0.$ **(05)**

c. Show that the function $y(x) = xe^x$ is a solution of the integral equation $y(x) = \sin x + 2\int_0^x \cos(x - t)\ y(t)\ dt.$ **(04)**

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Write Bessel equation and Hermite equation. **(02)**

b. Obtain the solution of $y(x) = 1 + \lambda\int_0^1 x\,t \cdot y(t)\ dt$ in the form $y(x) = 1 + \frac{3\lambda x}{2(3-\lambda)}\ (\lambda \neq 3).$ **(02)**

c. Reduce the differential equation $x^2y'' + xy' + (k^2x^2 - n^2)y = 0$ into Bessel's differential equation. **(02)**

d. Define: Separable Kernel. **(01)**

Q-5 **Attempt all questions** **(14)**

a. Solve: $y(x) = x + 2\int_0^x \cos(x - t)\ y(t)\ dt.$ **(06)**

b. Find the eigenvalues and eigenfunctions of the integral equation **(05)**

$$y(x) - \lambda\int_0^{2\pi} \sin x \sin t\ y(t)\ dt = 0.$$

c. State the Legendre differential equation and reduce it to Sturm-Liouville differential equation. **(03)**

OR

Q-5 **Attempt all questions**

a. Solve: $\frac{dy}{dx} = 3\int_0^x \cos 2(x - t)\ y(t)\, dt + 2$ given $y(0) = 1.$ **(06)**

b. Solve the Abel's integral equation$\int_0^x \frac{y(t)}{\sqrt{(x-t)}}\ dt = 1 + 2x - x^2.$ **(05)**

c. State the Laguerre differential equation and reduce it to Sturm-Liouville **(03)**

differential equation.

Q-6 **Attempt all questions** **(14)**

a. Solve the integral equation $y(x) = x + \lambda \int_0^1 (1 + x + t)\ y(t)\, dt$. **(07)**

b. Find the eigenvalues and the corresponding eigenfunctions of the differential equation $y'' + \lambda y = 0$ on the interval $[0, l]$ with the boundary conditions $y'(0) = 0$ and $y'(l) = 0$. **(07)**

OR

Q-6 **Attempt all Questions**

a. Discuss the eigenvalues and corresponding eigenfunctions for the integral equation $y(x) = F(x) + \lambda \int_0^1 (1 - 3xt)\, y(t)\ dt$. **(07)**

b. Find the eigenvalues and the corresponding eigenfunctions of the differential equation $4(e^{-x}y')' + (1 + \lambda)e^{-x}y = 0, y(0) = 0, y(1) = 0$. **(07)**